MINET Think Tank Event C
Measurement uncertainty when measuring the impossible
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UNCERTAINTY SOURCES

Giovanni Battista Rossi
Università degli Studi di Genova, DIMEC

Proposed approach

- Use foundational (i.e. discipline independent) arguments only
- Check whether this may apply to MtI problems also

Two (main) foundational aspects

- *What is the meaning of measurement?*
  - ⇒ The measurement scale
- *How do we measure?*
  - ⇒ The measurement process

The meaning of measurement/ the measurement scale

- To explain the meaning of measurement we need to refer to a more primitive notion: the notion of empirical relations
- Measurement allows the description of characteristics of objects by numbers in such a way as to reproduce in a numerical domain empirical relations
- For example as far as order relations are involved, the representation satisfies

$$a \succcurlyeq b \Leftrightarrow m(a) \geq m(b)$$

Is a deterministic description of empirical relations satisfactory?

- *Think to any measurable characteristic of your interest, that includes empirical order*
- Given any two objects, a, b, is it always possible to definitely establish whether $a \succ b$ or $a \sim b$ or $a \prec b$ holds?

Uncertainty in empirical relations 1

- If *a* and *b* are very “close” to each other, their “difference” is comparable with the “repeatability” of the comparator, and we repeat the comparison more times, we may observe sometimes $a \succ b$, some other $a \sim b$ and some other even $a \prec b$

Uncertainty in empirical relations 2

- If we have two equally reliable comparators, *C* and *D*, it may be that
  with C we obtain

  $a \succ_C b$

  whilst with D we obtain

  $a \sim_D b$

Possible interpretations

- Either $a \succ b$ or $a \sim b$ is true and one of the two comparators is wrong but we do not know which one
- The objects interact with the comparators in such a way that there are state changes in the two comparisons, but we are unable to define there states outside these comparisons

Probability as a logic for measurement

- Both cases above may be treated in probabilistic terms, by considering

$$a \succcurlyeq b$$

an uncertain statement and considering its probability

$$P\left(a \succcurlyeq b\right)$$

instead of its truth.

Probabilistic representation for order structures

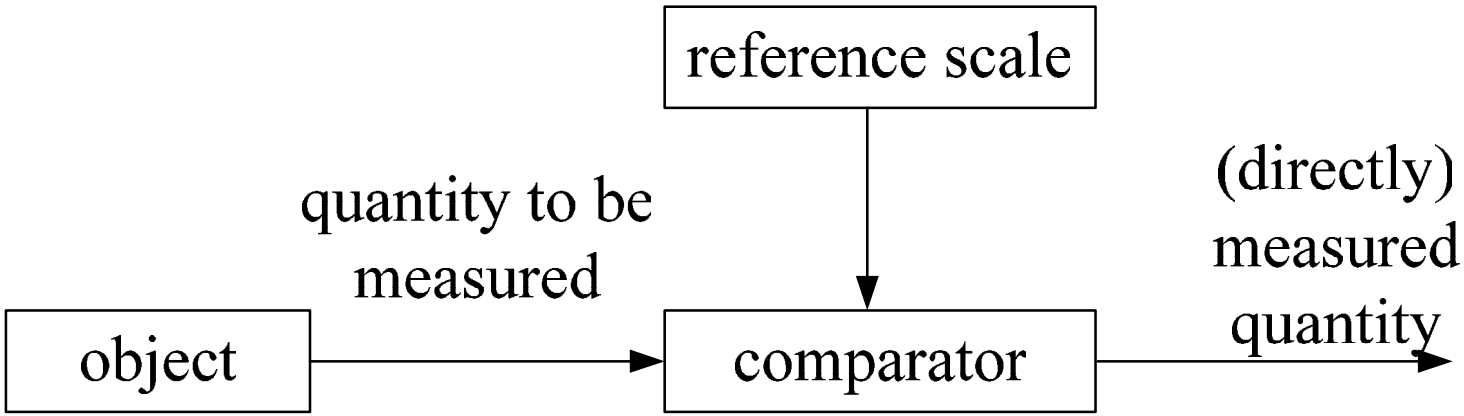
The representation theorem may be restated as

$$P\left(a \succcurlyeq b\right) = P\left(m\left(a\right) \geq m\left(b\right)\right)$$

The measurement process

How do we measure?

- we need a reference scale to be devised first
- and a way for comparing (directly or indirectly) unknown objects with the reference scale



Measurement process and measuring system

Measuring system

*an empirical system that is able to interact with objects carrying the quantity under investigation and to produce, as result of the interaction, **signs**, on the basis of which it is possible to assign a value to the object to measure*

Is a deterministic description of the measurement process satisfactory?

- *Think to any measurable characteristic of your interest*
- Given any object *a* is it always possible to definitely assign a measurement value $\hat{x}$

$$\hat{x} = \varphi(a)$$

to it?

Uncertainty in the measurement process

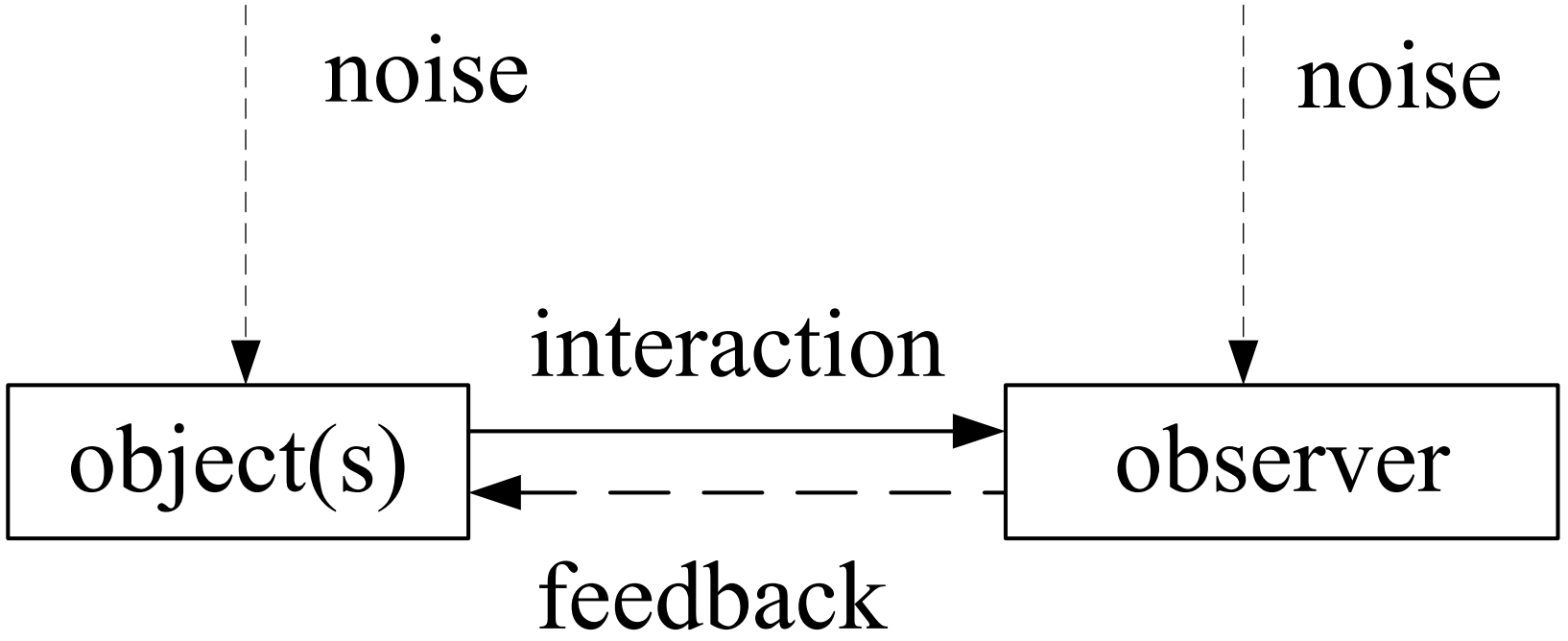
- If we repeat the measurement of the same object more times in conditions which are equivalent for the experimenter, we may obtain different values, $\hat{x}_1, \hat{x}_2, \ldots$
- If we measure the same objects through two equally reliable measurement processes, *M* and *N*, we may obtain (repeatedly) two different values,

$$\hat{x}_M = \varphi_M(a),\ \hat{x}_N = \varphi_N(a)$$

Again there may be different interpretations of these evidences (not to be mentioned now)

Information flux between the object (s) and the observer (comparator or measuring system) – ideal case

Information flux between the object (s) and the observer (comparator or measuring system) – Real case



A taxonomy of uncertainty sources

THREE MAIN CONCEPTUAL “COORDINATES”

- Related to empirical relations or to the measurement process
- Random variations versus systematic effects
- Information flux: object (s), observer or interaction